

A2 Further Mathematics Unit 6: Pure Mechanics B

General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations

The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only

MR = misread

PA = premature approximation

bod = benefit of doubt

oe = or equivalent

si = seen or implied

ISW = ignore subsequent working

F.T. = follow through (✓ indicates correct working following an error and ✗ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation

A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads

When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.

This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

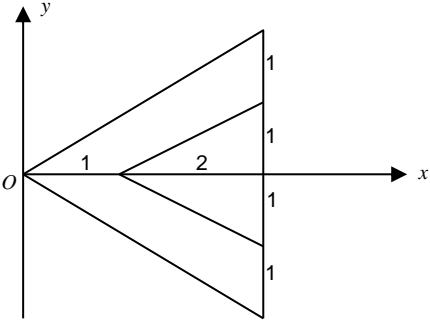
5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

A2 Further Mathematics Unit 6: Further Mechanics B

Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes	
1. (a)	N2L on ball, upwards positive $-0.01v^2 - 0.4g = 0.4a$	M1 A1	AO3 AO2	dim correct correct equation	
	$0.4v \frac{dv}{dx} = -3.92 - 0.01v^2$				
	$40v \frac{dv}{dx} = -(392 + v^2)$	A1	AO2	convincing	
	(b)	$40 \int \frac{v}{392 + v^2} dv = - \int dx$	M1	AO2	separate variables
		$20 \ln(392 + v^2) = -x + C$	A1 A1	AO1 AO1	$\ln(392 + v^2)$ everything correct
		When $t = 0, v = 17, x = 0$ $20 \ln(392 + 17^2) = C$ $C = 20 \ln(681)$	m1 A1	AO2 AO1	use of initial conditions
		$x = 20 \ln(681) - 20 \ln(392 + v^2)$			
		$x = 20 \ln \left(\frac{681}{392 + v^2} \right)$			
		$\frac{x}{20} = \ln \left(\frac{681}{392 + v^2} \right)$			
		$\left(\frac{681}{392 + v^2} \right) = e^{0.05x}$ $681 = (392 + v^2) e^{0.05x}$	m1	AO1	
	$v^2 = 681e^{-0.05x} - 392$				
	$v = \sqrt{681e^{-0.05x} - 392}$	A1	AO1		
(c)	At greatest height $v = 0$	M1	AO2		
	$x = 20 \ln \left(\frac{681}{392} \right) = 11.05$	A1	AO1	cao	
(d)	Speed of ball when it returns to O is less than 17 ms^{-1} .	B1	AO2		
	This is because energy is lost in overcoming air resistance.	E1	AO2		
		[14]			

Question Number	Solution	Mark	AO	Notes									
2 (b)	 <p data-bbox="347 674 746 707">Shape mass distance</p> <table data-bbox="347 745 746 987"> <tbody> <tr> <td data-bbox="347 745 427 819">C_1</td> <td data-bbox="443 745 619 819">$\frac{\pi}{3}(2)^2 \times 3\rho$</td> <td data-bbox="635 745 746 819">$\frac{3}{4} \times 3$</td> </tr> <tr> <td data-bbox="347 831 427 904">C_2</td> <td data-bbox="443 831 619 904">$\frac{\pi}{3} \times 1^2 \times 2\rho$</td> <td data-bbox="635 831 746 904">$1 + \frac{3}{4} \times 2$</td> </tr> <tr> <td data-bbox="347 916 427 987">Rem.</td> <td data-bbox="443 916 619 987">$\frac{\pi}{3}\rho(12-2)$</td> <td data-bbox="635 916 746 987">\bar{h}</td> </tr> </tbody> </table> <p data-bbox="347 1025 746 1059">Take moments about y axis</p> $\frac{\pi}{3}\rho \times 10 \times \bar{h} = \frac{\pi}{3} \times 12 \times \rho \times \frac{9}{4} - \frac{\pi}{3} \times 2\rho \times \frac{5}{2}$ $\bar{h} = \frac{11}{5}$	C_1	$\frac{\pi}{3}(2)^2 \times 3\rho$	$\frac{3}{4} \times 3$	C_2	$\frac{\pi}{3} \times 1^2 \times 2\rho$	$1 + \frac{3}{4} \times 2$	Rem.	$\frac{\pi}{3}\rho(12-2)$	\bar{h}	B1 B1 B1 M1 A1 A1	AO1 AO1 AO1 AO3 AO1 AO1	
C_1	$\frac{\pi}{3}(2)^2 \times 3\rho$	$\frac{3}{4} \times 3$											
C_2	$\frac{\pi}{3} \times 1^2 \times 2\rho$	$1 + \frac{3}{4} \times 2$											
Rem.	$\frac{\pi}{3}\rho(12-2)$	\bar{h}											

Question Number	Solution	Mark	AO	Notes
3. (a)	Using N2L	M1	AO3	
	$-0.2 - 0.03v = 9 \frac{dv}{dt}$	A1	AO2	
	$900 \frac{dv}{dt} = -(20 + 3v)$	A1	AO2	
(b)	$900 \int \frac{dv}{20+3v} = - \int dt$	M1	AO2	sep. var.
	$900 \times \frac{1}{3} \ln(20 + 3v) = -t (+ C)$	A1 A1	AO1 AO1	$\ln(20 + 3v)$ all correct
	When $t = 0, v = 20$ $C = 300 \ln 80$ Therefore	m1	AO2	used
	$t = 300 \ln(80) - 300 \ln(20 + 3v)$ $t = 300 \ln\left(\frac{80}{20 + 3v}\right)$	A1	AO1	
(c)	When body is at rest, $v=0$ $T = 300 \ln(80) - 300 \ln(20)$ $T = 300 \ln(4)$ $T = \underline{416 \text{ s}}$	m1 A1	AO2 AO1	used cao
		[10]		

GCE AS and A LEVEL FURTHER MATHEMATICS Sample Assessment Materials 70

Question Number	Solution	Mark	AO	Notes	
4. (a)	$\bar{x} = 4$ (cm)	B1	AO1		
(b)	Shape <i>GPQE</i>	mass 64	distance 4	B1 AO1	
	<i>EFG</i>	8π	$8 + \frac{16}{3\pi}$	B1 AO3	
	<i>APB</i>	π	$\frac{8}{3\pi}$		
	<i>CQD</i>	π	$\frac{8}{3\pi}$	B1 AO1	either <i>APB</i> or <i>CQD</i>
	<i>ABCDEFGF</i>	$64+6\pi$	\bar{y}	B1 AO1	areas
	Moments about <i>BC</i>			M1 AO3	
	$(64 + 6\pi)\bar{y} = 64 \times 4 + 8\pi \times \left(8 + \frac{16}{3\pi}\right) - 2\pi \times \frac{8}{3\pi}$			A1 AO1	
$\bar{y} = 5.967$ (cm) (correct to 3 d.p.)			A1 AO1		
(c)	If hanging in equilibrium, vertical passes through centre of mass.		M1 AO3	correct triangle	
	$\theta = \tan^{-1}\left(\frac{8-5.967}{4}\right)$		A1 AO1		
	$\theta = 26.94(1954\dots)^\circ$		A1 AO1		
			[11]		

GCE AS and A LEVEL FURTHER MATHEMATICS Sample Assessment Materials 71

Question Number	Solution	Mark	AO	Notes
5. (a)	$\mathbf{r}_A = 11\mathbf{i} + 6\mathbf{j} + (2\mathbf{i} + 7\mathbf{j})t$ $\mathbf{r}_B = 7\mathbf{i} + 10\mathbf{j} + (5\mathbf{i} + 4\mathbf{j})t$ If particles collide, $\mathbf{r}_A = \mathbf{r}_B$ for some value of t . For \mathbf{i} component $11 + 2t = 7 + 5t$ $t = \frac{4}{3}$ For \mathbf{j} component $6 + 7t = 10 + 4t$ $t = \frac{4}{3}$ Since the value for t for both components are equal, the particles collide. Conservation of momentum $m(2\mathbf{i} + 7\mathbf{j}) + 2m(5\mathbf{i} + 4\mathbf{j}) = 3m(x\mathbf{i} + y\mathbf{j})$ $12\mathbf{i} + 15\mathbf{j} = 3x\mathbf{i} + 3y\mathbf{j}$ $x = 4, y = 5$ $x\mathbf{i} + y\mathbf{j} = 4\mathbf{i} + 5\mathbf{j}$ (Ns)	M1 A1 M1 A1 A1 M1 A1 m1 A1	AO3 AO1 AO2 AO2 AO2 AO2 AO2 AO1	
(b)	$\mathbf{I} = \text{change in momentum}$ $\mathbf{I} = 2m(4\mathbf{i} + 5\mathbf{j}) - 2m(5\mathbf{i} + 4\mathbf{j})$ $\mathbf{I} = m(-2\mathbf{i} + 2\mathbf{j})$ $\mathbf{I} = 2m(-\mathbf{i} + \mathbf{j})$ (Ns)	M1 A1	AO3 AO1	used
(c)	$\text{Loss in KE} = \frac{1}{2}m(4 + 49) + \frac{1}{2}2m(25 + 16) - \frac{1}{2} \times 3m(16 + 25)$ $\text{Loss in KE} = 6m \text{ (J)}$	M1 A1	AO3 AO1	
		[13]		

Question Number	Solution	Mark	AO	Notes
6. (a)	At equilibrium $12g = \frac{\lambda \times 0.05}{0.75}$ $\lambda = \underline{1764 \text{ (N)}}$	M1 A1	AO3 AO1	use of Hooke's Law
(b)	Consider a displacement x from the equilibrium position. Apply N2L $12g - T = 12 \ddot{x}$ $12g - \frac{\lambda(0.05 + x)}{0.75} = 12 \ddot{x}$ $\ddot{x} = -(14)^2 x$ Therefore is SHM (with $\omega = 14$). Amplitude = <u>0.05 (m)</u> Period = $\frac{2\pi}{\omega} = \frac{\pi}{7}$ s	M1 A1 A1 B1 B1	AO3 AO3 AO2 AO1 AO1	ft λ
(c)	Maximum speed = $a\omega$ $= 0.05 \times 14$ $= \underline{0.7 \text{ (ms}^{-1}\text{)}}$	M1 A1	AO3 AO1	used ft a
(d)	Use of $v^2 = \omega^2(a^2 - x^2)$ with $\omega = 14$, $a = 0.05$ (c), $x = 0.03$ $v^2 = 14^2(0.05^2 - 0.03^2)$ $= 14^2 \times 0.04^2$ $v = \underline{0.56 \text{ (ms}^{-1}\text{)}}$	M1 A1 A1	AO3 AO2 AO1	ft a cao
(e)	Displacement from Origin = x $x = 0.05\cos(14t)$ When $t = 1.6$ $x = 0.05 \cos(14 \times 1.6)$ $x = (-)\underline{0.046 \text{ (m)}}$	M1 A1 A1	AO3 AO2 AO1	(Accept \pm) ft a (Accept \pm) cao
(f)	The seat is modelled as a particle. The spring is assumed to be light.	B1 B1	AO3 AO3	
		[17]		